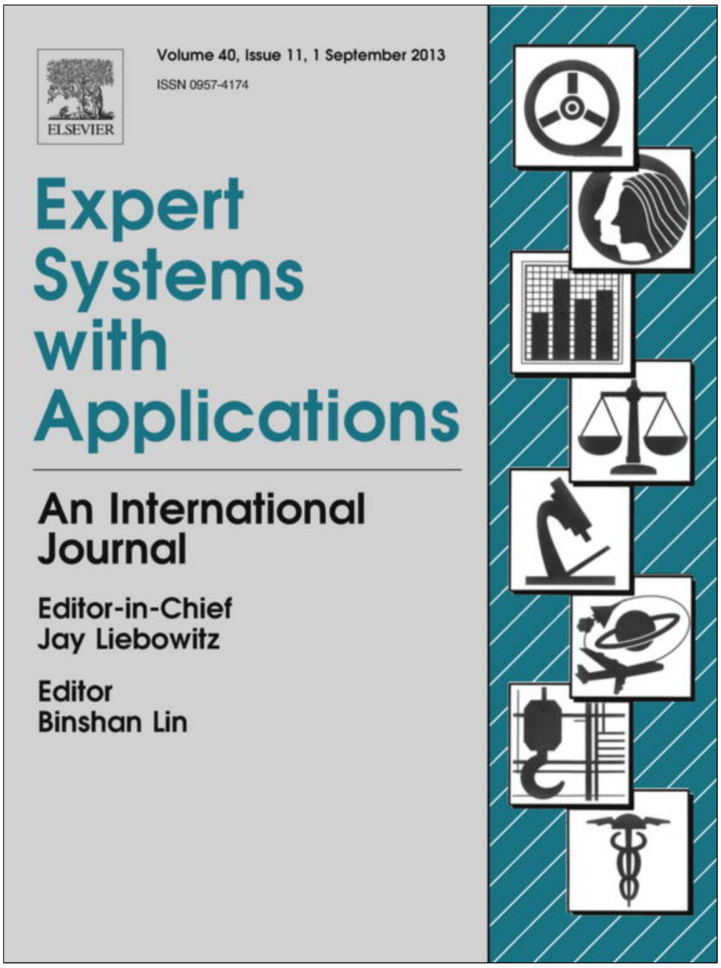
Volume 40, Issue 11, 1 September 2013
ISSN 0957-4174
ELSEVIER
Expert Systems with Applications
An International Journal
Editor-in-Chief
Jay Liebowitz
Editor
Binshan Lin

Expert Systems with Applications 40 (2013) 4503–4510

Contents lists available at SciVerse ScienceDirect

Expert Systems with Applications

journal homepage: www.elsevier.com/locate/eswa

Robust observer-based output feedback control for fuzzy descriptor systems

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ARTICLE INFO

Keywords:
Fuzzy descriptor systems
T–S fuzzy model
Linear matrix inequalities

ABSTRACT

This paper proposes a robust observer-based output feedback control for fuzzy descriptor systems. First, we represent singular nonlinear dynamic system into Takagi–Sugeno (T–S) fuzzy descriptor model which have a tighter representation for a wider class of nonlinear systems in comparison to general state-space models. To achieve the control objective, we design a fuzzy controller and observer in a unified and systematic manner. The stability analysis of the overall closed-loop fuzzy system leads to formulation of linear matrix inequalities (LMIs). The advantages of the approach are three fold. First, we consider conditions of immeasurable states which allows a practical design of sensorless control systems. Secondly, we address the robustness issue in the system which avoids control performance deterioration or instability due to disturbance or approximation errors in the system. Third, we formulate the overall control problem into LMIs. Using the observer and controller gains by solving LMIs, we carry out numerical simulations which verify theoretical statements.

1. Introduction

In the past decade fuzzy control has been proved to be very fruitful in many applications. Using the T–S fuzzy model (Takagi & Sugeno, 1985) representation of nonlinear systems into local linear fuzzy models has lead to vast amounts of research. For example fuzzy control (Chang, Chang, Tao, Lin, & Taur, 2012; Jain, Sivakumaran, & Radhakrishnan, 2011; Joh, Chen, & Langari, 1998; Precup, Radac, Tomescu, Petriu, & Preitl, 2013; Wang, Tanaka, & Griffin, 1996); fuzzy model based chaotic control and synchronization (Lian, Chiu, Chiang, & Liu, 2001b; Tanaka, Ikeda, & Wang, 1998b); robust fuzzy control and observer based approaches (Balasubramaniam, Vembarasan, & Rakkiyappan, 2012; Chiang & Liu, 2012; Chen, Tseng, & Uang, 1999, 2000; Lendek, Lauber, Guerra, Babuka, & Schutter, 2010; Lian, Chiu, Chiang, & Liu, 2001a; Soliman, Elshafei, Bendary, & Mansour, 2009; Sung, Kim, Park, & Joo, 2010; Tanaka, Ikeda, & Wang, 1996, 1998a; Tognetti, Oliveira, & Peres, 2012; Tsai, 2011; Tanaka & Sano, 1994) which take modeling errors, external disturbances, measurement errors into considerations. Many of the mentioned works approach the design of controllers and observers in an systematic manner. The stability analysis of the closed-loop system leads to formulation of linear matrix inequalities (LMIs) (Boyd, El Ghaoui, Feron, & Balakrishnan, 1994; Muralisankar, Gopalakrishnan, & Balasubramaniam, 2012). Then the controller and observer gains are found once the feasible LMIs are solved. The process of solving LMIs can be done numerically by powerful packaged software toolboxes (e.g., MATLAB LMI Toolbox).

On the other hand, descriptor systems have a tighter representation for a wider class of systems in comparison to traditional state-space models. This concept has also been extended to T–S fuzzy model descriptor systems (Chang & Yang, 2011; Taniguchi, Tanaka, & Wang, 2000). Note that using traditional T–S fuzzy modeling for Lagrangian mechanical systems, we need a fuzzy model representation for the inverse of the inertia matrix. This matrix inverse will drastically increase the rule numbers. On the other hand, if the fuzzy descriptor system is used, the number fuzzy rules will be decreased. This rule reduction is an important issue for LMI-based control synthesis since larger number of LMI rules may lead to infeasible problems.

In this paper, we extend the good properties of fuzzy descriptor systems and fuzzy observers into the design of robust output feedback control for fuzzy descriptor systems. The overall controller and observer design leads to formulating of LMIs. Then a multiple-stage process is utilized in place of simultaneously solving controller and observer parameters. The advantages of the approach are three fold. First, we consider conditions of immeasurable states which allows a practical design of sensorless control systems. Secondly, we address the robustness issue in the system which avoids control performance deterioration or instability due to disturbance and approximation errors. Third, we formulate the overall control problem into LMIs in a systemic and unified manner.

The rest of the paper is organized as follows. In Section 2, we introduce the fuzzy descriptor system representation of a singular nonlinear dynamic system. In Section 3, we carry out the stability

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http://dx.doi.org/10.1016/j.eswa.2013.01.053

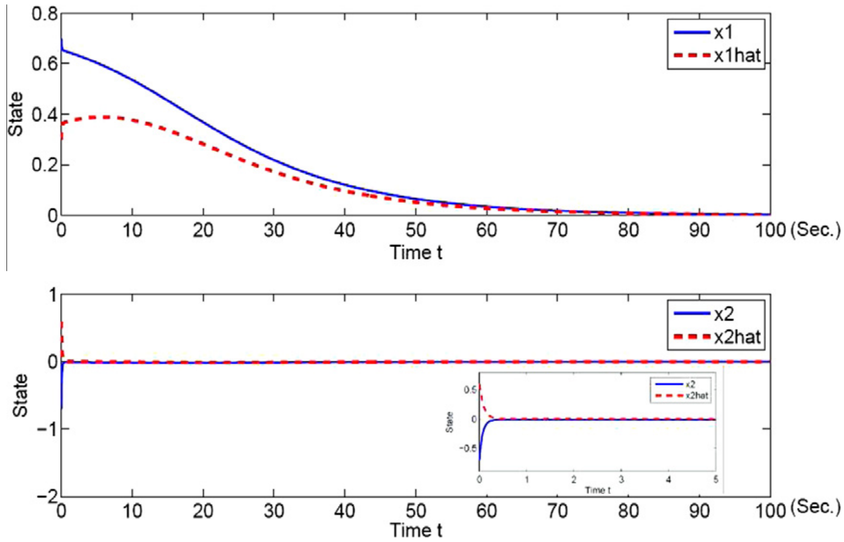

Fig. 1. State trajectories of descriptor system. State feedback: solid line; observer-based control: dotted line.

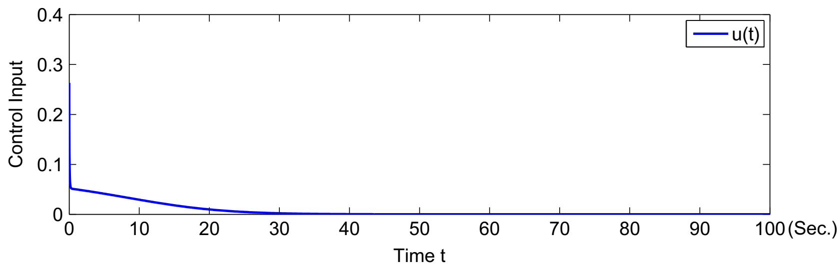

Fig. 2. Controller performance with the observer-based control approach.

analysis of the fuzzy descriptor system and formulate the LMI criterion. In Section 4, we carry out numerical simulations on the control design. Finally some conclusions are made in Section 5.

2. Preliminaries and problem formulation

A general singular nonlinear system is given as

$$
\begin{aligned}
&M(x(t))\dot{x}(t) = f(x(t)) + g(x(t))u(t) + \omega(t) \\
&y(t) = h(x)
\end{aligned}
\tag{1}
$$

where $x(t) = [x_1(t)\, x_2(t) \cdots x_n(t)]^T \in R^n$ is the state vector; $u(t) = [u_1(t)\, u_2(t) \cdots u_m(t)]^T \in R^m$ is the control input; $\omega(t)$ is the unknown but bounded disturbance; $M(x(t))$, $f(x(t))$, $g(x(t))$, $h(x(t))$ are smooth functions with $f(0) = 0$; and $y(t) \in R^q$ is the output. The T–S fuzzy representation of (1) is as follows:

Plant Rule k :
IF $z_1(t)$ is N_{k1} and $\cdots$ and $z_g(t)$ is N_{kg}
THEN
RHS Plant rule i :
IF $z_1(t)$is F_{i1} and $\cdots$ and $z_g(t)$ is F_{ig}
THEN $E_k\dot{x}(t) = A_i x(t) + B_i u(t) + \omega(t)$
$y(t) = C_i x(t)$

where N_{kg} and F_{ig} are fuzzy sets; $E_k \in R^{n\times n}$ is the descriptor matrix; $A_i \in R^{n\times n}$, $B_i \in R^{n\times m}$, $C_i \in R^{q\times n}$ are constant matrices with appropriate dimensions, and RHS stands for right-hand-side. The inferred output

$$
\begin{aligned}
&\sum_{k=1}^{r_e} \mu_k(z(t))E_k\dot{x}(t) = \sum_{i=1}^{r} \upsilon_i(z(t))\{A_i x(t) + B_i u(t) + \Delta f + \omega(t)\} \\
&y(t) = \sum_{i=1}^{r} \upsilon_i(z(t))C_i x(t) + \Delta h,
\end{aligned}
\tag{2}
$$

where $\mu_k(z(t)) = \frac{\alpha_k(z(t))}{\sum_{i=1}^{r_e}\alpha_k(z(t))}$, $\upsilon_i(z(t)) = \frac{\beta_i(z(t))}{\sum_{i=1}^{r}\beta_i(z(t))}$; $\alpha_k(z(t)) = \sum_{i=j}^{g} N_{kj}(z_j(t))$, $\beta_i(z(t)) = \sum_{i=j}^{g} F_{ij}(z_j(t))$; $N_{kj}(z_j(t))$, $F_{ij}(z_j(t))$ are the grade memberships of $z_j(t)$ in N_{kj}, F_{ij}, respectively; and $z(t) = [z_1(t)z_2(t) \cdots z_g(t)]$. It is straightforward that $\mu_k(z(t)) \geqslant 0$, $\sum_{k=1}^{r_e}\mu_k(z(t)) = 1$ and $\upsilon_i(z(t)) \geqslant 0$, $\sum_{i=1}^{r}\upsilon_i(z(t)) = 1$. We rewrite the fuzzy descriptor system (2) as

$$
\begin{aligned}
&E^*\dot{x}^*(t) = \sum_{i=1}^{r}\sum_{k=1}^{r_e} \upsilon_i(z(t))\mu_k(z(t))\{A^*_{ik}x^*(t) + B^*_i u(t) + \Delta f^* + \omega^*(t)\}, \\
&y(t) = \sum_{i=1}^{r} \upsilon_i(z(t))C^*_i x^*(t) + \Delta h
\end{aligned}
\tag{3}
$$

where $\Delta f = f(x(t)) - \sum_{i=1}^{r}\upsilon_i(z(t))A_i x(t)$, $\Delta h = h(x) - \sum_{i=1}^{r}\upsilon_i(z(t)) C_i\, x(t)$ are approximation errors. Define $x^*(t) = [x^T(t)\ \dot{x}^T(t)]^T$,

$$
E^* = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}, \quad A^*_{ik} = \begin{bmatrix} 0 & I \\ A_i & -E_k \end{bmatrix}, \quad B^*_i = \begin{bmatrix} 0 \\ B_i \end{bmatrix}, \quad C^*_i = [C_i \quad 0]
$$

$$
\Delta f^* = \begin{bmatrix} 0 \\ \Delta f \end{bmatrix}, \quad \omega^*(t) = \begin{bmatrix} 0 \\ \omega(t) \end{bmatrix}.
$$

If Δf^*, $\omega^*(t)$, Δh is omitted from (3), then we name the system as an "approximate system". On the other hand, (3) is the "true system".

The fuzzy descriptor system (3) is admissible Masubuchi, Kamitane, Ohara, and Suda (1997) if there exists $V(x^*(t)) = x^{*^T}(t)E^{*^T}Xx^*(t)$ and the following conditions are satisfied — (1) $\det(sE^* - \sum_{i=1}^{r}\sum_{i=1}^{r_e} v_i(z)\mu_k(z)A^*_{ik}) \neq 0$; (2) the open-loop system is impulse-free. Consequently, these conditions are satisfied if a common matrix $X \in R^{2n\times 2n}$, $\det X \neq 0$ such that $E^{*^T}X = X^TE^* \geqslant 0$ and $A^{*^T}_{ik}X + X^TA^*_{ik} < 0$.

First, we consider the open-loop system of (3) which is

$$E^*\dot{x}^*(t) = \sum_{i=1}^{r}\sum_{k=1}^{r_e} v_i(z(t))\mu_k(z(t))A^*_{ik}x^*(t) + \Delta f^* + \omega^*(t). \tag{4}$$

Second, we now design the controller rule as follows.

Control Rule i: IF $z_1(t)$ is F_{i1} and $\cdots$ and $z_g(t)$ is F_{ig} THEN

$$u(t) = -K^*_{ik}x^*(t) \quad \text{for } i = 1,2,\ldots,r.$$

where $K^*_{ik} = [K_{ik}\ 0]$ and K_{ik} are controller gains to be chosen later. We propose a modified PDC

$$u(t) = -\sum_{i=1}^{r}\sum_{k=1}^{r_e} v_i(z(t))\mu_k(z(t))K^*_{ik}x^*(t) \tag{5}$$

to stabilize the fuzzy descriptor system (3).

2.1. Immeasurable states

To estimate the immeasurable states, we design the observer rule as follows:

Plant Rule k :
IF $z_1(t)$is N_{k1} and $\cdots$ and $z_g(t)$ is N_{kg}
THEN
RHS Observer Rule i :
IF $z_1(t)$ is F_{i1} and $\cdots$ and $z_g(t)$ is F_{ig}
THEN $E_k\dot{\hat{x}}(t) = A_i\hat{x}(t) + B_iu(t) + L_i(y(t) - \hat{y}(t))$
$\hat{y}(t) = C_i\hat{x}(t)$

and L_i is the observer gain of the ith observer rule to be chosen later. The overall inferred output is

$$\begin{aligned}&\sum_{k=1}^{r_e}\mu_k(z(t))E_k\dot{\hat{x}}(t) = \sum_{i=1}^{r} v_i(z(t))\{A_i\hat{x}(t) + B_iu(t) + L_i[y(t) - \hat{y}(t)]\}\\&\hat{y}(t) = \sum_{i=1}^{r} v_i(z(t))C_i\hat{x}(t)\end{aligned} \tag{6}$$

where $z_1(t) \sim z_g(t)$ are the premise variables which consist of the states of the system; F_{ij} ($j = 1,2,\ldots,g$) are the fuzzy sets; r is the number of fuzzy rules; E_k, A_i, B_i and C_i are system matrices with appropriate dimensions. For simplicity, we assume that the membership functions have been normalized, i.e., $\sum_{i=1}^{r}\prod_{j=1}^{g}F_{ij}(z_j(t)) = 1$. Using the singleton fuzzifier, product inference, and weighted defuzzifier, the augmented fuzzy system is inferred as follows:

$$\begin{aligned}E^*\dot{\hat{x}}^*(t) &= \sum_{i=1}^{r}\sum_{k=1}^{r_e} v_i(z(t))\mu_k(z(t))\{A^*_{ik}\hat{x}^*(t) + B^*_iu(t) + L^*_{ik}[y(t) - \hat{y}(t)]\}\\&= \sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{k=1}^{r_e} v_i(z(t))v_j(z(t))\mu_k(z(t))\{A^*_{ik}\hat{x}^*(t)\\&\quad + B^*_iu(t)\ L^*_{ik}C^*_je^*(t) + L^*_{ik}\Delta h\Big\}\end{aligned}$$

$$\hat{y}(t) = \sum_{i=1}^{r} v_i(z(t))C^*_i\hat{x}^*(t), \tag{7}$$

where $L^*_{ik} = \left[0\ L^T_{ik}\right]^T$. Instead of (5), the PDC fuzzy controller

$$u(t) = -\sum_{i=1}^{r}\sum_{k=1}^{r_e} v_i(z(t))\mu_k(z(t))K^*_{ik}\hat{x}^*(t), \tag{8}$$

where $\hat{x}^*(t) = [\hat{x}^T(t)\ \dot{\hat{x}}^T(t)]^T$. Combining the fuzzy controller (8), fuzzy observer (7) and denoting $e^*(t) = x^*(t) - \hat{x}^*(t)$, $e^*(t) = [e^T(t)\ \dot{e}^T(t)]^T$, we arrive with the system representations:

$$\begin{aligned}E^*\dot{x}^*(t) &= \sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{k=1}^{r_e} v_i(z(t))v_j(z(t))\mu_k(z(t))\\&\quad\left\{\left(A^*_{ik} - B^*_iK^*_{jk}\right)x^*(t) + B^*_iK^*_{jk}e^*(t) + \triangle f^* + \omega^*(t)\right\}\\E^*\dot{e}^*(t) &= \sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{k=1}^{r_e} v_i(z(t))v_j(z(t))\mu_k(z(t))\\&\quad\left\{\left(A^*_{ik} - L^*_{ik}C^*_j\right)e^*(t) + \triangle f^* + \omega^*(t) - L^*_{ik}\Delta h\right\}\end{aligned} \tag{9}$$

Assumption 1. There exists a known bounding matrix $\Delta\phi_f$ such that $\|\Delta f\| \leqslant \|\Delta\phi_f x(t)\|$.

From the assumption above, we have

$$\Delta f^{*^T}\Delta f^* = \Delta f^T\Delta f \leqslant (\Delta\phi_f x(t))^T(\Delta\phi_f x(t)) = \left(\Phi_f x^*(t)\right)^T\left(\Phi_f x^*(t)\right)$$

where $\Phi_f = [\Delta\phi_f\ 0]$. The following theorem gives the sufficient condition of stability for (4) and (9).

Assumption 2. There exist bounding matrices ϕ_A, ϕ_C such that $\|\Delta f\| \leqslant \|\phi_A e(t)\|$, $\|\sum_{i=1}^{r}\sum_{i=k}^{r} v_i(z(t))\quad \mu_k(z(t))\ L^*_{ik}\Delta h\| \leqslant \|\sum_{i=1}^{r}\sum_{i=k}^{r} v_i(z(t))\mu_k(z(t))L^*_{ik}\phi_C e(t)\|$ for all $e(t)$.

According to Assumption 2, we have

$$\Delta f^{*^T}\Delta f^* = \Delta f^T\Delta f \leqslant (\phi_A e(t))^T(\phi_A e(t)) = (\Phi_A e^*(t))^T(\Phi_A e^*(t))$$

where $\Phi_A = [\phi_A\ 0]$.

$$\begin{aligned}\Delta h^{*T}_{ik}\Delta h^*_{ik} &= \left(\sum_{i=1}^{r}\sum_{i=k}^{r} v_i(z(t))\mu_k(z(t))(z(t))L^*_{ik}\Delta h\right)^T\\&\quad\times\left(\sum_{i=1}^{r}\sum_{i=k}^{r} v_i(z(t))\mu_k(z(t))(z(t))L^*_{ik}\Delta h\right)\\&\leqslant \left(\sum_{i=1}^{r}\sum_{i=k}^{r} v_i(z(t))\mu_k(z(t))(z(t))L^*_{ik}\phi_C e(t)\right)^T\\&\quad\times\left(\sum_{i=1}^{r}\sum_{i=k}^{r} v_i(z(t))\mu_k(z(t))(z(t))L^*_{ik}\phi_C e(t)\right)\\&= \left(\sum_{i=1}^{r}\sum_{i=k}^{r} v_i(z(t))\mu_k(z(t))(z(t))\Phi_{ikC}e^*(t)\right)^T\\&\quad\times\left(\sum_{i=1}^{r}\sum_{i=k}^{r} v_i(z(t))\mu_k(z(t))(z(t))\Phi_{ikC}e^*(t)\right)\\&\leqslant \sum_{i=1}^{r}\sum_{i=k}^{r} v_i(z(t))\mu_k(z(t))(z(t))e^{*T}(t)\Phi^T_{ikC}\Phi_{ikC}e^*(t)\end{aligned}$$

where $\Phi_{ikC} = \left[L^*_{ik}\phi_C\ 0\right]$ for $i = 1,2,\ldots,r$. If $\omega(t)$, Δh, Δf are omitted from (9), we name the system as an "approximate error system."

3. Stability analysis

In details, we present the stability criterion for the open-loop system (4) in the following:

Theorem 1. *The open-loop approximate fuzzy descriptor system (4) (where Δf^* and $\omega^*(t)$ are omitted) is quadratically stable if there exists a common matrix X such that*

$$E^{*^T}X = X^T E^* \geqslant 0 \tag{10}$$
$$A_{ik}^{*^T}X + X^T A_{ik}^* < 0$$

Furthermore, if there exists a common matrix X and $Q \geqslant 0$ such that (10) and

$$\begin{bmatrix} A_{ik}^{*^T}X + X^T A_{ik}^* + \Phi_f^T\Phi_f + Q + X^T X & X \\ X^T & -\frac{1}{\rho^2}I \end{bmatrix} < 0 \tag{11}$$

are satisfied for all the pairs (i,k) except for pairs $\upsilon_i(z(t))\mu_k(z(t)) = 0$ for all $z(t)$, then the true system (4) has the following robust performance

$$\int_0^T x^{*^T}(\tau)Qx^*(\tau)d\tau \leqslant x^{*^T}(0)Qx^*(0) + \frac{1}{\rho^2}\int_0^T \|\omega^*(\tau)\|d\tau. \tag{12}$$

Proof 1. Choose the Lyapunov function candidate

$$V(x^*(t)) = x^{*^T}(t)E^{*^T}Xx^*(t).$$

The time derivative

$$\begin{aligned}
\dot V(x^*(t)) &= \dot x^{*^T}(t)E^{*^T}Xx^*(t) + x^{*^T}(t)E^{*^T}X\dot x^*(t) \\
&= \sum_{i=1}^{r}\sum_{k=1}^{re}\upsilon_i(z(t))\mu_k(z(t))x^{*^T}(t)\left(A_{ik}^{*^T}X + X^T A_{ik}^*\right)x^*(t) \\
&\quad + (\Delta f^* + \omega^*(t))^T Xx^*(t) + x^{*^T}(t)X^T(\Delta f^* + \omega^*(t)) \\
&\leqslant \sum_{i=1}^{r}\sum_{k=1}^{re}\upsilon_i(z(t))\mu_k(z(t))\eta^T(t)\begin{bmatrix} A_{ik}^{*^T}X + X^T A_{ik}^* & X^T \\ X & -\frac{1}{\rho^2}I \end{bmatrix}\eta(t) \\
&\quad + \frac{1}{\rho^2}\omega^{*^T}(t)\omega^*(t) + (\Phi_f x^*(t))^T(\Phi_f x^*(t)) \\
&\quad + x^{*^T}(t)X^T Xx^*(t) \\
&= \sum_{i=1}^{r}\sum_{k=1}^{re}\upsilon_i(z(t))\mu_k(z(t)) \\
&\quad \times \eta^T(t)\begin{bmatrix} A_{ik}^{*^T}X + X^T A_{ik}^* + \Phi_f^T\Phi_f + Q + X^T X & X^T \\ X & -\frac{1}{\rho^2}I \end{bmatrix}\eta(t) \\
&\quad + \frac{1}{\rho^2}\omega^{*^T}(t)\omega^*(t) - x^{*^T}(t)Qx^*(t) \\
&\leqslant -x^{*^T}(t)Qx^*(t) + \frac{1}{\rho^2}\omega^{*^T}(t)\omega^*(t)
\end{aligned} \tag{13}$$

where $\eta^T(t) = [x^{*T}(t)\ \omega^{*T}(t)]$. Integrating on both sides of (13) with respect to time, we obtain the robust property (12). □

Corollary 1. *Let Q = block-diag $\{Q_{11}, Q_{22}\}$ >0. The conditions (10) and (11) are satisfied if there exists feasible solutions to the following EVP maximize ρ^2 $\mathbf{S}_1,\mathbf{S}_3,\mathbf{M}_1$, subject to*

$$S_1 = S_1^T \geqslant 0 \tag{14}$$

$$\begin{bmatrix} \chi_{11} & \chi_{12} & S_1^T & S_3^T & S_1^T & S_3^T \\ \chi_{12}^T & \chi_{22} & 0 & S_1^T & 0 & S_1^T \\ S_1 & 0 & -\frac{1}{\rho^2}I & 0 & 0 & 0 \\ S_3 & S_1 & 0 & -\frac{1}{\rho^2}I & 0 & 0 \\ S_1 & 0 & 0 & 0 & -I & 0 \\ S_3 & S_1 & 0 & 0 & 0 & -I \end{bmatrix} < 0 \tag{15}$$

where

$$\chi_{11} = A_i^T S_3 + S_3 A_i + \Delta\phi_f^T\Delta\phi_f + Q_{11},$$
$$\chi_{12} = S_1^T + A_i^T S_1 - S_3^T E_k,$$
$$\chi_{22} = -E_k^T S_1 - S_1 E_k + Q_{22}.$$

Proof 2. Define

$$X = \begin{bmatrix} S_1 & 0 \\ S_3 & S_1 \end{bmatrix}.$$

Then rewrite $E^{*T}X = X^T E^* \geqslant 0$. The above inequality implies

$$E^{*^T}X = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} S_1 & 0 \\ S_3 & S_1 \end{bmatrix} = \begin{bmatrix} S_1 & 0 \\ 0 & 0 \end{bmatrix} \geqslant 0,$$

$$X^T E^* = \begin{bmatrix} S_1^T & S_3^T \\ 0 & S_1^T \end{bmatrix}\begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} S_1^T & 0 \\ 0 & 0 \end{bmatrix} \geqslant 0.$$

Therefore (14) is proven.

From (11) and using Schur complements, we have

$$\begin{bmatrix} A_{ik}^{*^T}X + X^T A_{ik}^* + \Phi_f^T\Phi_f + Q & X^T & X^T \\ X & -\frac{1}{\rho^2}I & 0 \\ X & 0 & -I \end{bmatrix} < 0$$

Then by definition of X, the LMI (15) is obtained. □

In the following, we discuss the case where the overall controller and observer is considered.

Theorem 2. *The fuzzy descriptor system (2) along with controller (8) and observer (6) forming the closed-loop system (9) is asymptotically stable, if there exist nonsingular matrices P and R, matrices Z_1, Z_3, R_1, R_3, M_{jk} and H_{ik}, and scalars γ, ρ, $\varepsilon_n > 0$, $n = 1,2,\ldots,8$ satisfying the following LMIs:*

$$Z_1^T = Z_1 > 0, \tag{16}$$

$$\begin{bmatrix} \phi_{11} & \phi_{12} & Z_1 & 0 & Z_1^T\Delta\phi_f^T \\ \phi_{12}^T & \phi_{22} & -Z_3 & Z_1 & 0 \\ Z_1^T & -Z_3^T & -\frac{1}{\rho^2}W_{11} & -\frac{1}{\rho^2}W_{12} & 0 \\ 0 & Z_1^T & -\frac{1}{\rho^2}W_{12}^T & -\frac{1}{\rho^2}W_{22} & 0 \\ \Delta\phi_f Z_1 & 0 & 0 & 0 & -\varepsilon_4 I \end{bmatrix} < 0, \quad i = 1,2,\ldots,r. \tag{17}$$

$$\begin{bmatrix} \tilde{\phi}_{11} & \tilde{\phi}_{12} & 2Z_1 & 0 & 2Z_1^T\Delta\phi_f^T \\ \tilde{\phi}_{12}^T & \tilde{\phi}_{22} & -2Z_3 & 2Z_1 & 0 \\ 2Z_1^T & -2Z_3^T & -\frac{2}{\rho^2}W_{11} & -\frac{2}{\rho^2}W_{12} & 0 \\ 0 & 2Z_1^T & -\frac{2}{\rho^2}W_{12}^T & -\frac{2}{\rho^2}W_{22} & 0 \\ 2\Delta\phi_f Z_1 & 0 & 0 & 0 & -2\varepsilon_4 I \end{bmatrix} < 0, \quad i<j. \tag{18}$$

$$R_1^T = R_1 > 0, \tag{19}$$

$$\begin{bmatrix} \Upsilon_{11} & \Upsilon_{12} \\ \Upsilon_{12}^T & \Upsilon_{22} \end{bmatrix} < 0, \quad i = 1,2,\ldots,r. \tag{20}$$

$$\begin{bmatrix} \widetilde{\Upsilon}_{11} & \widetilde{\Upsilon}_{12} \\ \widetilde{\Upsilon}_{12}^T & \widetilde{\Upsilon}_{22} \end{bmatrix} < 0, \quad i<j. \tag{21}$$

where matrices are denoted as

$$\Upsilon_{11} = \begin{bmatrix} \lambda_{11} & \lambda_{12} & R_1^T & R_3^T & 0 & \phi_{c1}^T L_{ik}^T \\ \lambda_{12}^T & \lambda_{22} & 0 & R_1^T & 0 & \phi_{c2}^T L_{ik}^T \\ R_1 & 0 & -\frac{1}{\rho^2}I & 0 & 0 & 0 \\ R_3 & R_1 & 0 & -\frac{1}{\rho^2}I & 0 & 0 \\ 0 & 0 & 0 & 0 & -\varepsilon_8 I & 0 \\ L_{ik}\phi_{c1} & L_{ik}\phi_{c2} & 0 & 0 & 0 & -\varepsilon_8 I \end{bmatrix},$$

$$\Upsilon_{12} = \begin{bmatrix} \phi_A^T & K_{ik}^T & \varepsilon_8 R_1^T & \varepsilon_8 R_3^T & \varepsilon_6 R_1^T & \varepsilon_6 R_3^T \\ 0 & 0 & 0 & \varepsilon_8 R_1 & 0 & \varepsilon_6 R_1^T \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\Upsilon_{22} = diag[-\varepsilon_6 I,\ -\varepsilon_1 I,\ -\varepsilon_8 I,\ -\varepsilon_8 I,\ -\varepsilon_6 I,\ -\varepsilon_6 I],$$

$$\widetilde{\Upsilon}_{11} = \begin{bmatrix} \tilde{\lambda}_{11} & \tilde{\lambda}_{12} & 2R_1^T & 2R_3^T & 0 & \phi_{c1}^T L_{jk}^T & 0 & \phi_{c1}^T L_{ik}^T \\ \tilde{\lambda}_{12}^T & \tilde{\lambda}_{22} & 0 & 2R_1^T & 0 & \phi_{c2}^T L_{jk}^T & 0 & \phi_{c2}^T L_{ik}^T \\ 2R_1 & 0 & -\frac{2}{\rho^2}I & 0 & 0 & 0 & 0 & 0 \\ 2R_3 & 2R_1 & 0 & -\frac{2}{\rho^2}I & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\varepsilon_7 I & 0 & 0 & 0 \\ L_{jk}\phi_{c1} & L_{jk}\phi_{c2} & 0 & 0 & 0 & -\varepsilon_7 I & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\varepsilon_5 I & 0 \\ L_{ik}\phi_{c1} & L_{ik}\phi_{c2} & 0 & 0 & 0 & 0 & 0 & -\varepsilon_5 I \end{bmatrix},$$

$$\widetilde{\Upsilon}_{12} = \begin{bmatrix} \phi_A^T & \varepsilon_7 R_1^T & \varepsilon_7 R_3^T & \varepsilon_5 R_1^T & \varepsilon_5 R_3^T & \varepsilon_6 R_1^T & \varepsilon_6 R_3^T & K_{jk}^T & K_{ik}^T \\ 0 & 0 & \varepsilon_7 R_1^T & 0 & \varepsilon_5 R_1^T & 0 & \varepsilon_6 R_1^T & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\widetilde{\Upsilon}_{22} = diag\left[-\frac{1}{2}\varepsilon_6 I,\ -\varepsilon_7 I,\ -\varepsilon_7 I,\ -\varepsilon_5 I,\ -\varepsilon_5 I,\ -\frac{1}{2}\varepsilon_6 I,\ -\frac{1}{2}\varepsilon_6 I,\right. \\ \left. -\varepsilon_2 I,\ -\varepsilon_3 I\right],$$

$$\begin{aligned}
\phi_{11} &= -Z_3^T - Z_3 + \widetilde{Q}_{a1} + \varepsilon_4 I, \\
\phi_{12} &= Z_1^T A_i^T - M_{ik}^T B_i^T + Z_3^T E_k^T + Z_1, \\
\phi_{22} &= -Z_1^T E_k^T - E_k Z_1 + \widetilde{Q}_{a2} + \varepsilon_1 B_i B_i^T + \varepsilon_4 I, \\
\tilde{\phi}_{11} &= -2Z_3^T - 2Z_3 + 2\varepsilon_4 I + 2\widetilde{Q}_{a1}, \\
\tilde{\phi}_{12} &= Z_1^T A_i^T - M_{jk}^T B_i^T + Z_1^T A_j^T - M_{ik}^T B_j^T + 2Z_3^T E_k^T + 2Z_1, \\
\tilde{\phi}_{22} &= -2E_k Z_1 - 2Z_1^T E_k^T + 2\widetilde{Q}_{a2} + 2\varepsilon_4 I + \varepsilon_2 B_i B_i^T + \varepsilon_3 B_j B_j^T, \\
\lambda_{11} &= \gamma A_i^T R_1 - \gamma C_i^T H_{ik}^T + \gamma R_1^T A_i - \gamma H_{ik} C_i + S_{a1}, \\
\lambda_{12} &= A_i^T R_1 - C_i^T H_{ik}^T + R_1^T - \gamma R_1^T E_k, \\
\lambda_{22} &= -E_k^T R_1 - R_1^T E_k + S_{a2}, \\
\tilde{\lambda}_{11} &= \gamma A_i^T R_1 - \gamma C_j^T H_{ik}^T + \gamma A_j^T R_1 - \gamma C_i^T H_{jk}^T + \gamma R_1^T A_i - \gamma H_{ik} C_j \\
&\quad + \gamma R_1^T A_j - \gamma H_{jk} C_i + 2S_{a1}, \\
\tilde{\lambda}_{12} &= A_i^T R_1 - C_j^T H_{ik}^T + A_j^T R_1 - C_i^T H_{jk}^T + 2R_1^T - 2\gamma R_1^T E_k, \\
\tilde{\lambda}_{22} &= -2E_k^T R_1 - 2R_1^T E_k + 2S_{a2}.
\end{aligned}$$

The controller and observer gains are accordingly $K_{jk} = M_{jk}Z_1^{-1}$ and $L_{ik} = R_1^{-1}H_{ik}$, if there exists a common matrix $Q > 0$ and $S > 0$, the system (9) has the following robust performance

$$\int_0^T x^{*T}(\tau)Qx^*(\tau)d\tau \leqslant x^{*T}(0)E^{*^T}Px^*(0) + \frac{1}{\rho^2}\int_0^T \|\omega^*(\tau)\|d\tau$$

$$\int_0^T e^{*T}(\tau)Se^*(\tau)d\tau \leqslant e^{*T}(0)E^{*^T}Re^*(0) + \frac{1}{\rho^2}\int_0^T \|\omega^*(\tau)\|d\tau \tag{22}$$

Proof 3. Define

$$P = \begin{bmatrix} S_1 & 0 \\ S_3 & S_1 \end{bmatrix}, \quad Q = \begin{bmatrix} Q_{a1} & 0 \\ 0 & Q_{a2} \end{bmatrix}, \quad R = \begin{bmatrix} R_1 & 0 \\ R_3 & R_1 \end{bmatrix}, \quad S = \begin{bmatrix} S_{a1} & 0 \\ 0 & S_{a2} \end{bmatrix},$$

$$\psi^T(t) = [x^{*T}(t)\ e^{*T}(t)].$$

Then, we rewrite $E^{*T}P = P^T E^* \geqslant 0$ as $P^{-T}\ E^{*T} = E^* P^{-1} \geqslant 0$ and $E^{*T}R = R^T E^* \geqslant 0$. The above inequality implies

$$\begin{bmatrix} S_1 & 0 \\ S_3 & S_1 \end{bmatrix}^{-T}\begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} S_1 & 0 \\ S_3 & S_1 \end{bmatrix}^{-1} \geqslant 0.$$

We then arrive with

$$\begin{bmatrix} Z_1^T & -Z_3^T \\ 0 & Z_1^T \end{bmatrix}\begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} Z_1 & 0 \\ -Z_3 & Z_1 \end{bmatrix} = \begin{bmatrix} Z_1 & 0 \\ 0 & 0 \end{bmatrix} \geqslant 0,$$

where $Z_1 = S_1^{-1}$ and $Z_3 = S_1^{-1}S_3S_1^{-1}$. Note that

$$\begin{bmatrix} S_1 & 0 \\ S_3 & S_1 \end{bmatrix}\begin{bmatrix} Z_1 & 0 \\ -Z_3 & Z_1 \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix}.$$

We consider the Lyapunov function candidate

$$V(\psi(t)) = \sum_{i=1}^{2} V_i(\psi(t)) \tag{23}$$

where

$$V_1(x^*(t)) = x^{*T}(t)E^{*T}Px^*(t), \quad V_2(e^*(t)) = e^{*T}(t)E^{*T}Re^*(t).$$

Therefore the time derivative

$$\dot{V}_1(x^*(t)) = \dot{x}^{*T}(t)E^{*T}Px^*(t) + x^{*T}(t)E^{*T}P\dot{x}^*(t),$$
$$\dot{V}_2(e^*(t)) = \dot{e}^{*T}(t)E^{*T}Re^*(t) + e^{*T}(t)E^{*T}R\dot{e}^*(t).$$

Therefore the time derivative along (9) is

$$\begin{aligned}\dot{V}_1(x^*(t)) \leqslant \sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{k=1}^{r_e}&\upsilon_i(z(t))\upsilon_j(z(t))\mu_k(z(t))\\ &\times\Big\{x^{*T}(t)\left(G_{ijk}^TP + P^TG_{ijk}\right)x^*(t) + e^{*T}(t)\left(B_i^*K_{jk}^*\right)^TPx^*(t)\\ &+ x^{*T}(t)P^T\left(B_i^*K_{jk}^*\right)e^*(t) + (\Delta f^{*T} + \omega^{*T}(t))Px^*(t)\\ &+ x^{*T}(t)P^T(\Delta f^* + \omega^*(t)) + x^{*T}(t)Qx^*(t) - x^{*T}(t)Qx^*(t)\\ &+ \frac{1}{\rho^2}\omega^{*T}(t)\omega^*(t) - \frac{1}{\rho^2}\omega^{*T}(t)\omega^*(t)\Big\}\end{aligned}$$

which further leads to

$$\begin{aligned}\dot{V}_1(x^*(t)) \leqslant \sum_{i=1}^{r}\sum_{k=1}^{r_e}&\upsilon_i^2(z(t))\mu_k(z(t))\\ &\times\Big\{x^{*T}(t)\left(G_{iik}^TP + P^TG_{iik}\right)x^*(t) + e^{*T}(t)(B_i^*K_{ik}^*)^TPx^*(t)\\ &+ x^{*T}(t)P^T(B_i^*K_{ik}^*)e^*(t) + (\Delta f^{*T} + \omega^{*T}(t))Px^*(t)\\ &+ x^{*T}(t)P^T(\Delta f^* + \omega^*(t)) + x^{*T}(t)Qx^*(t) - x^{*T}(t)Qx^*(t)\\ &+ \frac{1}{\rho^2}\omega^{*T}(t)\omega^*(t) - \frac{1}{\rho^2}\omega^{*T}(t)\omega^*(t)\Big\}\\ &+ 2\sum_{i=1}^{r}\sum_{i<j}\sum_{k=1}^{r_e}\upsilon_i(z(t))\upsilon_j(z(t))\mu_k(z(t))\\ &\times\Big\{x^{*T}(t)\left(\frac{(G_{ijk}+G_{jik})^T}{2}P + P^T\frac{(G_{ijk}+G_{jik})}{2}\right)x^*(t)\\ &+ e^{*T}(t)\frac{\left(B_i^*K_{jk}^* + B_j^*K_{ik}^*\right)^T}{2}Px^*(t) + x^{*T}(t)P^T\frac{\left(B_i^*K_{jk}^* + B_j^*K_{ik}^*\right)}{2}e^*(t)\\ &+ \left(\Delta f^{*T} + \omega^{*T}(t))Px^*(t) + x^{*T}(t)P^T(\Delta f^* + \omega^*(t)\right)\\ &+ x^{*T}(t)Qx^*(t) - x^{*T}(t)Qx^*(t) + \frac{1}{\rho^2}\omega^{*T}(t)\omega^*(t)\\ &- \frac{1}{\rho^2}\omega^{*T}(t)\omega^*(t)\Big\}\end{aligned}$$

According to inequality $2x^Ty \leqslant \varepsilon x^Tx + \varepsilon^{-1}y^Ty$, where $\varepsilon > 0$, we have

$$\begin{aligned}&e^{*^T}(t)(B_i^*K_{ik}^*)^TPx^*(t) + x^{*^T}(t)P^T(B_i^*K_{ik}^*)e^*(t)\\ &\quad\leqslant \varepsilon_1x^{*^T}(t)P^TB_i^*B_i^{*^T}Px^*(t) + \varepsilon_1^{-1}e^{*^T}(t)K_{ik}^{*^T}K_{ik}^*e^*(t),\quad i=j.\end{aligned}$$

$$\begin{aligned}&e^{*^T}(t)\frac{\left(B_i^*K_{jk}^* + B_j^*K_{ik}^*\right)^T}{2}Px^*(t) + x^{*^T}(t)P^T\frac{\left(B_i^*K_{jk}^* + B_j^*K_{ik}^*\right)}{2}e^*(t)\\ &\quad\leqslant \varepsilon_2x^{*^T}(t)P^T\frac{B_i^*B_i^{*T}}{2}Px^*(t) + \varepsilon_2^{-1}e^{*^T}(t)\frac{K_{jk}^{*T}K_{jk}^*}{2}e^*(t) + \varepsilon_3x^{*^T}(t)P^T\\ &\qquad\times\frac{B_j^*B_j^{*T}}{2}Px^*(t) + \varepsilon_3^{-1}e^{*^T}(t)\frac{K_{ik}^{*T}K_{ik}^*}{2}e^*(t),\quad i\\ &\quad< j.\end{aligned}$$

and

$$\begin{aligned}&(\Delta f^{*T} + \omega^{*T}(t))Px^*(t) + x^{*T}(t)P^T(\Delta f^* + \omega^*(t))\\ &\quad\leqslant \varepsilon_4^{-1}\Delta f^{*T}\Delta f^* + \varepsilon_4x^{*^T}(t)P^TPx^*(t) + \omega^{*^T}(t)Px^*(t) + x^{*^T}(t)P^T\omega^*(t).\end{aligned}$$

We therefore have

$$\begin{aligned}\dot{V}_2(e^*(t)) \leqslant \sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{k=1}^{r_e}&\upsilon_i(z(t))\upsilon_j(z(t))\\ &\times\Big\{\left[\left(A_{ik}^* - L_{ik}^*C_j^*\right)e^*(t) - L_{ik}^*\Delta h + \Delta f^* + \omega^*(t)\right]^TRe^*(t)\\ &+ e^{*^T}(t)R^T\left[\left(A_{ik}^* - L_{ik}^*C_j^*\right)e^*(t) - L_{ik}^*\Delta h + \Delta f^* + \omega^*(t)\right]\\ &+ e^{*T}(t)Se^*(t) - e^{*T}(t)Se^*(t) + \frac{1}{\rho^2}\omega^{*T}(t)\omega^*(t)\\ &- \frac{1}{\rho^2}\omega^{*T}(t)\omega^*(t)\Big\}\end{aligned}$$

which further leads to

$$\begin{aligned}\dot{V}_2(e^*(t)) \leqslant \sum_{i=1}^{r}\sum_{k=1}^{r_e}&\upsilon_i^2(z(t))\mu_k(z(t))\Big\{e^{*T}(t)\left(W_{iik}^TR + R^TW_{iik}\right)e^*(t)\\ &+ \Delta h_{ik}^{*T}Re^*(t) + e^{*T}(t)R^T\Delta h_{ik}^* + (\Delta f^{*T} + \omega^{*T}(t))Re^*(t)\\ &+ e^{*T}(t)R^T(\Delta f^* + \omega^*(t)) + e^{*T}(t)Se^*(t) - e^{*T}(t)Se^*(t)\\ &+ \frac{1}{\rho^2}\omega^{*T}(t)\omega^*(t) - \frac{1}{\rho^2}\omega^{*T}(t)\omega^*(t)\Big\}\\ &+ 2\sum_{i=1}^{r}\sum_{i<j}\sum_{k=1}^{r_e}\upsilon_i(z(t))\upsilon_j(z(t))\mu_k(z(t))\\ &\times\Big\{e^{*T}(t)\left(\frac{(W_{ijk}+W_{jik})^T}{2}R + R^T\frac{(W_{ijk}+W_{jik})}{2}\right)e^*(t)\\ &+ e^{*T}(t)R^T\frac{\left(\Delta h_{ik}^* + \Delta h_{jk}^*\right)}{2} + \frac{\left(\Delta h_{ik}^* + \Delta h_{jk}^*\right)^T}{2}Re^*(t)\\ &+ \left(\Delta f^{*T} + \omega^{*T}(t))Re^*(t) + e^{*T}(t)R^T(\Delta f^* + \omega^*(t)\right)\\ &+ e^{*T}(t)Se^*(t) - e^{*T}(t)Se^*(t) + \frac{1}{\rho^2}\omega^{*T}(t)\omega^*(t)\\ &- \frac{1}{\rho^2}\omega^{*T}(t)\omega^*(t)\Big\}.\end{aligned}$$

where $G_{ijk} = A_{ik}^* - B_i^*K_{jk}^*,\ W_{ijk} = A_{ik}^* - L_{ik}^*C_j^*, \Delta h_{ik}^* = -\sum_{i=1}^{r}\sum_{k=1}^{r_e}\upsilon_i(z(t))\mu_k(z(t))L_{ik}^*\Delta h$, with inequalities

$$\begin{aligned}\Delta h_{ik}^{*T}Re^*(t) + e^{*T}(t)R^T\Delta h_{ik}^* &\leqslant \varepsilon_8^{-1}\Delta h_{ik}^{*T}\Delta h_{ik}^* + \varepsilon_8e^{*T}(t)R^TRe^*(t), i\\ &= j,\quad e^{*T}(t)R^T\frac{\left(\Delta h_{ik}^* + \Delta h_{jk}^*\right)}{2}\\ &\quad+ \frac{\left(\Delta h_{ik}^* + \Delta h_{jk}^*\right)^T}{2}Re^*(t)\\ &\leqslant \varepsilon_5^{-1}\frac{\Delta h_{ik}^{*T}\Delta h_{ik}^*}{2} + \varepsilon_7^{-1}\frac{\Delta h_{jk}^{*T}\Delta h_{jk}^*}{2} + (\varepsilon_5\\ &\quad+ \varepsilon_7)e^{*T}(t)\frac{R^TR}{2}e^*(t),\quad i\ j\end{aligned}$$

and

$$\begin{aligned}&(\Delta f^{*T} + \omega^{*T}(t))Re^*(t) + e^{*T}(t)R^T(\Delta f^* + \omega^*(t))\\ &\quad\leqslant \varepsilon_6^{-1}\Delta f^{*T}\Delta f^* + \varepsilon_6x^{*^T}(t)R^TRx^*(t) + \omega^{*^T}(t)Rx^*(t) + x^{*^T}(t)R^T\omega^*(t).\end{aligned}$$

From the Assumption 1 and 2 above, we have

$$\begin{aligned}\dot{V}(\psi(t)) \leqslant & \sum_{i=1}^{r}\sum_{k=1}^{r_e}\upsilon_i^2(z(t))\mu_k(z(t))\{\xi^T(t)\Lambda_1\xi(t)-x^{*T}(t)Qx^*(t)\\&+\frac{1}{\rho^2}\omega^{*T}(t)\omega^*(t)\Big\}+2\sum_{i=1}^{r}\sum_{i<j}\sum_{k=1}^{r_e}\upsilon_i(z(t))\upsilon_j(z(t))\mu_k(z(t))\\&\times\Big\{\xi^T(t)\Lambda_2\xi(t)-x^{*T}(t)Qx^*(t)+\frac{1}{\rho^2}\omega^{*T}(t)\omega^*(t)\Big\}\\&+\sum_{i=1}^{r}\sum_{k=1}^{r_e}\upsilon_i^2(z(t))\mu_k(z(t))\{\zeta^T(t)\Lambda_3\zeta(t)-e^{*T}(t)Se^*(t)\\&+\frac{1}{\rho^2}\omega^{*T}(t)\omega^*(t)\Big\}+2\sum_{i=1}^{r}\sum_{i<j}\sum_{k=1}^{r_e}\upsilon_i(z(t))\upsilon_j(z(t))\mu_k(z(t))\\&\times\Big\{\zeta^T(t)\Lambda_4\zeta(t)-e^{*T}(t)Se^*(t)+\frac{1}{\rho^2}\omega^{*T}(t)\omega^*(t)\Big\}\\ \leqslant & -x^{*^T}(t)Qx^*(t)-e^{*^T}(t)Se^*(t)+\frac{2}{\rho^2}\omega^{*^T}(t)\omega^*(t)\end{aligned} \tag{24}$$

where $\xi^T(t) = [x^{*T}(t)\ \omega^{*T}(t)]$, $\zeta^T(t) = [e^{*T}(t)\ \omega^{*T}(t)]$, and

$$\Lambda_1=\begin{bmatrix} G_{iik}^TP+P^TG_{iik}+Q+\varepsilon_1P^TB_i^*B_i^{*^T}P+\varepsilon_4^{-1}\Phi_f^T\Phi_f+\varepsilon_4P^TP & P^T\\ P & -\frac{1}{\rho^2}I\end{bmatrix},$$

$$\Lambda_2=\begin{bmatrix}\begin{pmatrix}(G_{ijk}+G_{jik})^TP+P^T(G_{ijk}+G_{jik})+2Q+2\varepsilon_4^{-1}\Phi_f^T\Phi_f\\ +2\varepsilon_4P^TP+\varepsilon_2P^TB_i^*B_i^{*^T}P+\varepsilon_3P^TB_i^*B_i^{*^T}P\end{pmatrix} & 2R^T\\ 2R & -\frac{2}{\rho^2}I\end{bmatrix},$$

$$\Lambda_3=\begin{bmatrix}\begin{pmatrix}W_{iik}^TR+R^TW_{iik}+S+\varepsilon_1^{-1}K_{ik}^{*^T}K_{ik}^*+\varepsilon_8^{-1}\Phi_{ikC}^T\Phi_{ikC}\\ +(\varepsilon_6+\varepsilon_8)R^TR+\varepsilon_6^{-1}\Phi_A^T\Phi_A\end{pmatrix} & R^T\\ R & -\frac{1}{\rho^2}I\end{bmatrix}.$$

$$\Lambda_4=\begin{bmatrix}\begin{pmatrix}(W_{ijk}+W_{jik})^TR+R^T(W_{ijk}+W_{jik})+2S+\varepsilon_5^{-1}\Phi_{ikC}^T\Phi_{ikC}\\ +\varepsilon_7^{-1}\Phi_{jkC}^T\Phi_{jkC}+\varepsilon_6^{-1}\Phi_A^T\Phi_A+\varepsilon_2^{-1}K_{jk}^{*^T}K_{jk}^*+\varepsilon_3^{-1}K_{ik}^{*^T}K_{ik}^*\\ +(\varepsilon_5+2\varepsilon_6+\varepsilon_7)R^TR\end{pmatrix} & 2R^T\\ 2R & -\frac{2}{\rho^2}I\end{bmatrix}.$$

Integrating on both sides of (24) with respect to time, we obtain the robust property (22). Therefore, when $E^{*T}P = P^TE^* \geqslant 0$, $E^{*T}R = R^TE^* \geqslant 0$, $\Lambda_1 < 0$, $\Lambda_2 < 0$, $\Lambda_3 < 0$, $\Lambda_4 < 0$ the stability and the closed-loop system (9) is proven. We multiply the inequality $\Lambda_1 < 0$ and $\Lambda_2 < 0$ by the matrix $diag[P^{-T}, P^{-T}]$ and its transpose on the left and right, respectively. Then, we set

$$P^{-T}=\begin{bmatrix}Z_1^T & -Z_3^T\\ 0 & Z_1^T\end{bmatrix}=\widetilde{P}$$

where $Z_1 > 0$. Define new variables $\widetilde{Q}_{a1}=\widetilde{P}Q_{a1}\widetilde{P}^T,\ \widetilde{Q}_{a2}=\widetilde{P}Q_{a2}\widetilde{P}^T$,

$$W=\widetilde{P}\widetilde{P}^T=\begin{bmatrix}W_{11} & W_{12}\\ W_{12}^T & W_{22}\end{bmatrix}>0$$

and by Schur complement, the inequalities $\Lambda_1 < 0$ and $\Lambda_2 < 0$ are equivalent to (16)–(18), which $M_{jk} = K_{jk}Z_1$ (or $M_{ik} = K_{ik}Z_1$). From the feasible solutions of (19)–(21), substitute $K_{jk}=M_{jk}Z_1^{-1}$ (or $K_{ik}=M_{ik}Z_1^{-1}$), and scalars $\varepsilon_1\sim\varepsilon_4$, into the inequality $\Lambda_3 < 0$, $\Lambda_4 < 0$. Let $H_{ik}=R_1^TL_{ik}$, $R_3 = \gamma R_1$, where $R_1 > 0$. Then we have $\Lambda_3 < 0$ and $\Lambda_4 < 0$, which are equivalent to (19)–(21) by the Schur complement. This completes the proof of the theorem. □

Since the simultaneous solution of observer gains in (19)–(21) is not trivial, we utilize the multiple-step method to cope with the problem. In the first step, the following observer inequality is equal to the following LMI

$$\begin{bmatrix}\lambda_{11} & \lambda_{12} & R_1^T & R_3^T\\ \lambda_{12}^T & \lambda_{22} & 0 & R_1^T\\ R_1 & 0 & -\frac{1}{\rho^2}I & 0\\ R_3 & R_1 & 0 & -\frac{1}{\rho^2}I\end{bmatrix}<0, \tag{25}$$

$$\begin{bmatrix}\tilde{\lambda}_{11} & \bar{\lambda}_{12} & 2R_1^T & 2R_3^T\\ \bar{\lambda}_{12}^T & \bar{\lambda}_{22} & 0 & 2R_1^T\\ 2R_1 & 0 & -\frac{2}{\rho^2}I & 0\\ 2R_3 & 2R_1 & 0 & -\frac{2}{\rho^2}I\end{bmatrix}<0. \tag{26}$$

From 19 and 25, 26, we are able to solve $H_{ik}(H_{jk})$, R_1 and then obtain $L_{ik}(L_{jk})$. In the second step, the observer parameters are substituted into (19)–(21), we the solve the remaining unknown parameters in (19)–(21). Based on the analysis above, we are summarized as follows:

Step 1. Give suitable bounding matrices $\Delta\phi_f$, ϕ_A, ϕ_{c1}, ϕ_{c2} and scalars $\varepsilon_1\sim\varepsilon_8$ in advance.
Step 2. Solve the LMIPs in (16)–(18) to obtain $Z_1,\ Z_3,\widetilde{Q}_{a1},\widetilde{Q}_{a2},\ W$ and $K_{ik}(K_{jk})$.
Step 3. Solve the LMIP in (25) and (26) to obtain $H_{ik}(H_{jk})$, R_1 and $L_{ik}(L_{jk})$ (note that these are not the final solutions).
Step 4. Substitute $K_{ik}(K_{jk})$, $L_{ik}(L_{jk})$ into (19)–(21) to obtain R_1, S_{a1}, S_{a2} and $L_{ik}(L_{jk})$.
Step 5. Substitute gains from Step 1, 2 and 4 into (16)–(21).
Step 6. Check whether (16)–(21) is a negative definite matrix. If not, go back to Step 1.
Step 7. Output $K_{ik}(K_{jk})$, $L_{ik}(L_{jk})$, ρ.

4. Numerical simulations

We carry out numerical simulations on the following example to verify the theoretical derivations. Consider a nonlinear system (Taniguchi et al., 2000)

$$(1+a\cos\theta(t))\ddot{\theta}(t)=-b\dot{\theta}^3(t)+c\theta(t)+du(t),$$

where the range of $\dot{\theta}(t)$ as $|\dot{\theta}(t)|<\phi_1$.

Considering output feedback case with immeasurable states, the observer descriptor system (6) as the form

$$\begin{aligned}\sum_{k=1}^{2}\mu_k(z(t))E_k\dot{\hat{x}}(t)=&\sum_{i=1}^{2}\upsilon_i(z(t))\{A_i\hat{x}(t)+A_{hi}\hat{x}(t-h(t))+B_iu(t)\\&+L_i[y(t)-\hat{y}(t)]\}\end{aligned}$$

$$\hat{y}(t)=\sum_{i=1}^{2}\upsilon_i(z(t))C_i\hat{x}(t)$$

where

$$E_1=\begin{bmatrix}1 & 0\\ 0 & 1+a\end{bmatrix},\quad E_2=\begin{bmatrix}1 & 0\\ 0 & 1-a\end{bmatrix},\quad A_1=\begin{bmatrix}0 & 1\\ c & -b\phi_1^2\end{bmatrix},$$

$$A_2=\begin{bmatrix}0 & 1\\ c & -b\phi_2^2\end{bmatrix},\quad B_1=B_2=\begin{bmatrix}0\\ d\end{bmatrix},\quad C_1=C_2=[0.1\quad 1],$$

$$\Delta\phi_f=[0.1\quad 0.1],\quad \phi_A=[0.01\quad 0.01],$$

$$\phi_{c1}=[0.02\quad 0.02],\quad \phi_{c2}=[0.03\quad 0.03].$$

We let $a = 0.2$, $b = 1$, $c = -1$, $d = 10$, $\phi_1 = 4$, $\phi_2 = 0$, $\varepsilon_1 = 4$, $\varepsilon_2 = 1.2$, $\varepsilon_3 = 1.2$, $\varepsilon_4 = 1.1$, $\varepsilon_5 = 0.2$, $\varepsilon_6 = 0.1$, $\varepsilon_7 = 0.3$, $\varepsilon_8 = 0.2$, $\gamma = 1.4$ and $\rho = 0.35$. The observer membership functions are defined as

$\mu_1(\hat{x}_1(t)) = \frac{1+\cos\hat{x}_1(t)}{2}$, $\mu_2(\hat{x}_1(t)) = \frac{1-\cos\hat{x}_1(t)}{2}$, $\upsilon_1(\hat{x}_2(t)) = \frac{\hat{x}_2^2(t)}{2}$, $\upsilon_2(\hat{x}_2(t)) = 1 - \frac{\hat{x}_2^2(t)}{2}$. According to LMIs (16)–(21), we can obtain control gains K_{jk} and observer gains L_{ik} separately where $K_{11} = [0.5833\ -0.8304]$, $K_{12} = [0.3829\ -1.0396]$, $K_{21} = [0.5833\ 0.7696]$, $K_{22} = [0.3829\ 0.5604]$ $L_{11} = [-0.4288\ -2.0061]^T$, $L_{12} = [-0.3544\ -2.1465]^T$, $L_{21} = [-0.4096\ 0.6630]^T$, $L_{22} = [-0.3125\ 0.5497]^T$.

The Figs. 1 and 2 show the convergence result under the observer-based control law

$$u(t) = -\sum_{i=1}^{r}\sum_{k=1}^{r_e}\upsilon_i(z(t))\mu_k(z(t))K_{ik}^*\hat{x}^*(t)$$

with initial condition $x(0) = [0.7\ -0.7]^T$ and $\hat{x}(0) = [0.3\ 0.6]^T$.

5. Conclusions

We have proposed an robust observer-based output feedback control for fuzzy descriptor systems in presence of immeasurable states, approximation errors and uncertainty. The observer and controller design has been implemented in a unified and systematic manner where gains are solved by a set of LMIPs. Numerical simulation results verify the theoretical claims.

Acknowledgment

This work was supported by National Science Council, R.O.C., under Grant NSC 101-2221-E-032-023.

References

Balasubramaniam, P., Vembarasan, V., & Rakkiyappan, R. (2012). Delay-dependent robust asymptotic state estimation of takagivsugeno fuzzy hopfield neural networks with mixed interval time-varying delays. *Expert Systems with Applications, 39*, 472–481.

Boyd, S., El Ghaoui, L., Feron, E., & Balakrishnan, V. (1994). *Linear matrix inequalities in system and control theory volume 15 of studies in applied mathematics*. Philadelphia, PA: SIAM.

Chang, X.-H., & Yang, G.-H. (2011). A descriptor representation approach to observer-based control synthesis for discrete-time fuzzy systems. *Fuzzy Sets and Systems, 185*, 38–51.

Chang, Y.-H., Chang, C.-W., Tao, C.-W., Lin, H.-W., & Taur, J.-S. (2012). Fuzzy sliding-mode control for ball and beam system with fuzzy ant colony optimization. *Expert Systems with Applications, 39*, 3624–3633.

Chen, B.-S., Tseng, C.-S., & Uang, H.-J. (1999). Robustness design of nonlinear dynamic systems via fuzzy linear control. *IEEE Transactions on Fuzzy Systems, 7*, 571–585.

Chen, B.-S., Tseng, C.-S., & Uang, H.-J. (2000). Mixed H_2/H_∞; fuzzy output feedback control design for nonlinear dynamic systems: An LMI approach. *IEEE Transactions on Fuzzy Systems, 8*, 249–265.

Chiang, T.-S., & Liu, P. (2012). Robust output tracking control for discrete-time nonlinear systems with time-varying delay: Virtual fuzzy model LMI-based approach. *Expert Systems with Applications, 39*, 8239–8247.

Jain, R., Sivakumaran, N., & Radhakrishnan, T. (2011). Design of self tuning fuzzy controllers for nonlinear systems. *Expert Systems with Applications, 38*, 4466–4476.

Joh, J., Chen, Y.-H., & Langari, R. (1998). On the stability issues of linear takagi-sugeno fuzzy models. *IEEE Transactions on Fuzzy Systems, 6*, 402–410.

Lendek, Z., Lauber, J., Guerra, T., Babuka, R., & Schutter, B. D. (2010). Adaptive observers for T–S fuzzy systems with unknown polynomial inputs. *Fuzzy Sets and Systems, 161*, 2043–2065.

Lian, K.-Y., Chiu, C.-S., Chiang, T.-S., & Liu, P. (2001a). Secure communications of chaotic systems with robust performance via fuzzy observer-based design. *IEEE Transactions on Fuzzy Systems, 9*, 212–220.

Lian, K.-Y., Chiu, C.-S., Chiang, T.-S., & Liu, P. (2001b). LMI-based fuzzy chaotic synchronization and communications. *IEEE Transactions on Fuzzy Systems, 9*, 539–553.

Masubuchi, I., Kamitane, Y., Ohara, A., & Suda, N. (1997). $H\infty$ control for descriptor systems: A matrix inequalities approach. *Automatica, 33*, 669–673.

Muralisankar, S., Gopalakrishnan, N., & Balasubramaniam, P. (2012). An LMI approach for global robust dissipativity analysis of T–S fuzzy neural networks with interval time-varying delays. *Expert Systems with Applications, 39*, 3345–3355.

Precup, R.-E., Radac, M.-B., Tomescu, M. L., Petriu, E. M., & Preitl, S. (2013). Stable and convergent iterative feedback tuning of fuzzy controllers for discrete-time SISO systems. *Expert Systems with Applications, 40*, 188–199.

Soliman, M., Elshafei, A., Bendary, F., & Mansour, W. (2009). LMI static output-feedback design of fuzzy power system stabilizers. *Expert Systems with Applications, 36*, 6817–6825.

Sung, H. C., Kim, D. W., Park, J. B., & Joo, Y. H. (2010). Robust digital control of fuzzy systems with parametric uncertainties: LMI-based digital redesign approach. *Fuzzy Sets and Systems, 161*, 919–933.

Takagi, T., & Sugeno, M. (1985). Fuzzy identification of systems and its applications to modeling and control. *IEEE Transactions on Systems, Man, and Cybernetics, 15*, 116–132.

Tanaka, K., Ikeda, T., & Wang, H. (1996). Robust stabilization of a class of uncertain nonlinear systems via fuzzy control: Quadratic stabilizability, H infinity control theory, and linear matrix inequalities. *IEEE Transactions on Fuzzy Systems, 4*, 1–13.

Tanaka, K., Ikeda, T., & Wang, H. (1998a). Fuzzy regulators and fuzzy observers: Relaxed stability conditions and LMI-based designs. *IEEE Transactions on Fuzzy Systems, 6*, 250–265.

Tanaka, K., Ikeda, T., & Wang, H. (1998b). A unified approach to controlling chaos via an LMI-based fuzzy control system design. *IEEE Transactions on Circuits and Systems I: Fundamental Theory and Applications, 45*, 1021–1040.

Tanaka, K., & Sano, M. (1994). A robust stabilization problem of fuzzy control systems and its application to backing up control of a truck-trailer. *IEEE Transactions on Fuzzy Systems, 2*, 119–134.

Taniguchi, T., Tanaka, K., & Wang, H. (2000). Fuzzy descriptor systems and nonlinear model following control. *IEEE Transactions on Fuzzy Systems, 8*, 442–452.

Tognetti, E. S., Oliveira, R. C., & Peres, P. L. (2012). Reduced-order dynamic output feedback control of continuous-time T–S fuzzy systems. *Fuzzy Sets and Systems, 207*, 27–44.

Tsai, S.-H. (2011). Robust H_∞ control for van de vusse reactor via T–S fuzzy bilinear scheme. *Expert Systems with Applications, 38*, 4935–4944.

Wang, H., Tanaka, K., & Griffin, M. (1996). An approach to fuzzy control of nonlinear systems: Stability and design issues. *IEEE Transactions on Fuzzy Systems, 4*, 14–23.